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RANKING, UNEMPLOYMENT
DURATION, AND WAGES

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Abstract

Firms often receive multiple acceptable applications for vacancies, requiring a choice among candidates. This paper contrasts equilibria when firms select workers at random and when firms select the worker with the shortest spell of unemployment, called ranking. With the filling of vacancies unaffected by the selection rule, both equilibria have the same aggregate dynamics, but different distributions of unemployment durations. With the threat point for the Nash bargained wage being a worker with zero unemployment duration, the wage with ranking is much more sensitive to changes in the tightness of the labor market. The same holds for efficiency wages.

This paper develops a model of the joint determination of unemployment, the distribution of unemployment durations, and wages. The model is based on two central assumptions. The first is that most vacancies receive multiple applications. The second is that when firms receive multiple acceptable applications, they hire the worker who has been unemployed for the least amount of time. We refer to this second assumption as "ranking" and contrast it throughout to the assumption of random hiring, or "no-ranking". We show that these assumptions have a number of important implications. First, the exit rate from unemployment is a decreasing function of duration, with the effect of duration being stronger the higher the rate of unemployment. Second, the wage depends on the distributions of prospective unemployment durations; one implication is that long term unemployment, per se, has little effect on wages. Third, looking at the relation between the wage and the aggregate level of unemployment, and comparing ranking to the case of no-ranking, the wage moves less with the level of unemployment, but more with the change in unemployment. We see those implications as consistent with stylized facts, consistent in particular with characterizations of European unemployment in the 1980's.

We focus on ranking in the labor market as we see it as a relevant and important characteristic of labor markets. A caveat is in order. We shall discuss available empirical evidence below, when we have stated our assumptions explicitly. But it is clear that firms do not rank only on the basis of unemployment duration; that perceptions of differential quality among applicants also affect hiring. To the extent that quality comes from matching skills and

job needs, the hiring process will reflect both ranking and no-ranking elements, so that ranking and no-ranking can be seen as the two polar cases. It is also clear that ranking is not the only reason why the exit rate decreases with the duration of unemployment. Composition effects are certainly at work, with the most attractive and qualified workers leaving the unemployment pool faster. Duration dependence, either through loss of skills, or decreasing search intensity also accounts for some of the decline in individual exit rates. Our motivation for focusing on ranking to the exclusion of these other aspects is the usual one, that it allows for a better understanding of the implications of this assumption.

The paper is organized as follows. Section I characterizes the matching process and derives a matching function. A particularly convenient characterization for our purposes is as an urn-ball process, with vacancies as urns and applications by unemployed workers as balls. The distribution of applications per vacancy depends on aggregate labor market conditions, thus on aggregate unemployment and vacancies. Vacancies with at least one acceptable application are filled. When a vacancy receives more than one acceptable application, firms use a rule to choose among applicants. Under those assumptions, the rule determines who is hired but does not affect the number of hires.

Section II embodies the matching function derived in section I in a model of the labor market. As in our previous papers, we formalize the labor market as a market with continual job creation and destruction, and thus labor reallocation. Having done so, we can characterize the equilibrium values of unemployment, vacancies and hires.

Section III derives the dynamics of the distributions of unemployment durations and of exit rates. It characterizes steady state distributions, those distributions which obtain when unemployment, vacancies, and hires are constant. It shows how the exit rate depends on duration, and how this dependence is itself a function of the state of the labor market. In a tight labor market, a long term unemployed worker may be the only applicant at a given vacancy. In a very depressed labor market, most vacancies receive many applications, so that the probability of being hired decreases quickly with duration.

Section IV derives the behavior of wages. It does so under two assumptions. The first is one of bilateral Nash bargaining between the selected applicant and the hiring firm. The second is the impossibility of precommitment, or bonding, so that workers can renegotiate once they have been hired. The implication is that the wage depends on the labor market prospects of those who are employed, were they to become unemployed, rather than on the prospects of the average unemployed worker. Thus, for example, in a labor market which is depressed but picking up, prospects for those becoming unemployed will be significantly better than for those who have been unemployed for a while; in turn this will generate substantial pressure on wages. Section IV focuses on the steady state relation between unemployment and wages. Dynamics require simulations and this is done in the next section.

Using a discrete time version of the model, section V presents dynamic simulations of the joint behavior of unemployment, unemployment durations, exit rates, and wages. It compares the behavior of wages under the assumptions of ranking and no-ranking. The simulations show how, in the case of ranking, anticipations of a decrease in unemployment have a strong effect on the wage even in a currently depressed labor market.

The qualitative nature of the results holds for other theories that have wages depend on the market prospects of the employed workers. In particular, this is shown in Section VI which repeats the analysis of sections IV and V for an efficiency wage which satisfies a no-shirking condition.

Section VII discusses an implication and several issues in the model.

Section I. The matching process

Our formalization of the matching process as an urn-ball process follows Gerald Butters (1977) and Robert Hall (1979). We present first a static version, and then the continuous time extension which is used in the rest of the paper.

1. A static urn-ball model.

Consider an economy with V vacancies and U unemployed workers. Firms which want to fill a job post a vacancy. Think of the vacancies as urns. Each worker makes one acceptable application with probability a, no application otherwise; applications are submitted at random to one of the vacancies. The parameter a captures in a very rough way a number of dimensions of the matching process. It reflects the intensity of search by workers and firms, as well as the skill and geographical distributions of workers and jobs. Think of the aU applications as balls.

If the numbers of vacancies and applicants are large, the binomial distribution giving the distribution of applications at a given vacancy can be approximated by a Poisson distribution. The probability that a vacancy has no

application is given by $\exp[-aU/V]$. Thus the number of aggregate hires, which is equal to the number of vacancies which receive one or more applications, is equal to:

(1) $$H = V(1-\exp[-aU/V]).$$

This is the basic matching function of our model. In a more realistic model, the application rate, a, would depend on the state of labor market; workers are more likely to learn of vacancies when there are more of them. The probability of making an acceptable application would also vary across workers, according to both their characteristics and their unemployment durations. We ignore those aspects here.

2. A continuous time version.

While one can build a discrete time model in which hires are given each period by equation (1), we prefer to work with a continuous time version of the initial model. While the derivation of the matching function is initially more intricate, the payoff is the usual one of better tractability. The trusting reader can go directly to equation (2) below.

Consider a model where vacancies are posted for one period, say a week. At the end of the week, the application window is closed and the applications generated by the vacancy are counted. If no applications have been received, the vacancy is posted for another week. If one or more applications have been received, the vacancy is filled[1].

The week is divided in intervals of length $1/n$. Vacancies are evenly staggered, so that V/n vacancies start in each interval, each vacancy lasting for a week. Applications are also evenly staggered over each interval. Let A_n be the number of applications during an interval of length $1/n$; the relation of A_n to U will be derived later. We assume that a worker can have only one application pending at any time[2].

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1 We assume that the vacancy window is of fixed length, although this policy is not generally optimal; a complicated time and state dependent stopping rule which would be optimal in realistic settings is unlikely to add sufficiently to the realism of the model to justify the analytical complexity.

2 To see the complexities being avoided, consider the case where workers can apply to many vacancies simultaneously. Upon hearing of their hiring by one firm, workers would withdraw their other applications. Thus, instead of a pure birth process for applications, we would have to allow for a birth-death process. With the ranking assumption, multiple applications would increase duration dependence.

The number of hires per interval is equal to the number of vacancies closing with at least one application and is thus given by:

$$H_n = (V/n)(1-\exp[-nA_n/V]).$$

In each interval, V/n application windows are closed. The term in brackets gives the probability that the vacancy has received at least one application while it was open and is therefore filled. At any time, there are V vacancies open. Each of the A_n applications in any given interval has an equal chance of landing in each vacancy and each vacancy remains open for n intervals, i.e., one week.

What remains to be determined is A_n. Let a'/n be the probability[3] that an unemployed worker with no application pending makes an application during any interval of length 1/n. Let X be the pool of workers with applications pending. Thus, applications are equal to a'/n times the pool of unemployed workers without applications pending, U-X:

$$A_n = (a'/n)(U-X).$$

Consider in turn the expression for X: only 1/n of the vacancies which were open in the last interval have closed, so that a proportion (n-1)/n of the applications made during the last interval is still pending. Of the applications made 2 intervals ago, a proportion (n-2)/n is still pending, and so on. This gives the expression for X:

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[3] The assumption that applications are made at rate a'/n per interval with a' independent of n is inessential. What is essential is that, as we let n tend to infinity, the rate of applications per interval of length 1/n converges to some constant, say a", divided by n.

$$X = ((n-1)/n + (n-2)/n + \ldots + 1/n))A_n.$$

Solving for A_n and taking the limit as n goes to infinity gives:

$$nA_n = aU,$$

where

$$a = a'/(1+a'/2).$$

Replacing nA_n in the matching function, and noting that the flow per week is $H_n n$, gives:

(2) $$h = V(1-\exp[-aU/V]),$$

where h is the instantaneous flow of hires. This matching function is the continuous time counterpart to equation (1) and holds when each vacancy is opened for a discrete length of time, and vacancies are uniformly staggered. For use below, note that, given the assumption that vacancies are posted for one week, V stands for both the stock of vacancies and the flow of newly posted vacancies at one point in time[4].

We shall use equation (2) to study both steady state relations and dynamics. Note however that, outside of steady state, the equation holds only as an

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[4] The assumption that vacancies last for one period (week) implicitly defines the period. An alternative - but formally equivalent - formalization strategy would be to have a basic period of fixed length, say a week, and have vacancies last for 1/b weeks. Following the same steps as in the text gives:

$h = bV(1-\exp[-aU/bV])$, where $a = a'/(1+(a'/2b))$.

Although we shall not do so in the paper, it is interesting to consider the effects of varying b. As b becomes large, vacancies last for shorter and shorter periods of time, reducing the probability of multiple applications. If, in the case of multiple applications, firms use the length of unemployment as a screening device, a lower probability of multiple applications increases the likelihood of being hired for the long term unemployed.

approximation. This is because its derivation assumed that applications are made at a constant rate during the week. Outside of steady state, applications will not be constant within the week and equation (2) holds only as an approximation. The approximation will be better the shorter the length of time during which a vacancy is open and the smoother the change in application rates[5]
.

Below, we will use the Poisson distribution of applications per vacancy and the implied matching function, (2), in a continuous time model. In doing so, we will ignore other effects of the finiteness of the window. In particular, we will ignore the possibility of bad productivity shocks while vacancy windows are open. We will also ignore the implications of the finite windows for the time shape of the probability of finding a job and for the discrete nature of the time profile of filling a vacancy. That is, we shall use the finiteness of the window only to derive the distribution of multiple applications, not recognizing any other effects from the presence of windows.

Section II. A model of the labor market; flows and stocks

Having specified the matching process, we need to embed it in a model of the labor market. We use the same minimalist model we have used in an earlier paper

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[5] This issue does not arise in the dynamic simulations presented in sections V and VI. In those sections, the model simulated is a discrete time version of the model with no overlapping vacancies.

(1989), a model which captures the constant process of job creation and destruction which characterizes labor markets[6].

There are K jobs in the economy. To produce a revenue of y, a job must be productive and filled with one worker. If either unproductive or/and unfilled, it produces 0. Productivity for each job follows a Markov process in continuous time. A productive job becomes unproductive with flow probability π_0. An unproductive job becomes productive with flow probability π_1. At any point in time, some jobs become productive, some jobs become unproductive. This is the mechanism we use to capture the flows of job creation and job destruction that exist in the economy.

Thus, at any point in time, some jobs are productive and filled, some are productive but unfilled, in which case a vacancy is posted, and some are unproductive and thus also unfilled. We denote the three stocks respectively by E (as one job requires one worker, E is also employment), V (for vacancies), and I (for idle capacity). From the definitions, $K = E+V+I$. At any point in time, some workers are employed, and some are unemployed. We denote those stocks by E and U respectively. The labor force $L=E+U$ is assumed fixed.

These assumptions imply that the behavior of the labor market is characterized by a system of two differential equations:

(3) $$dE(t)/dt = h(t) - \pi_0 E(t),$$

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6 The version we use here is stripped of details inessential for our current purpose. In particular, we do not allow for quits and new entrants. The reader is refered to our previous paper for a number of extensions.

$$dV(t)/dt = -h(t) - \pi_0 V(t) + \pi_1 I(t),$$

where, from the previous section, hires are given by:

$$h(t) = V(t)(1-\exp[-aU(t)/V(t)]).$$

The flow from employment to unemployment is equal to the flow of filled jobs becoming unproductive. The flow from unemployment to employment is in turn equal to hires. Hires depend on unemployment and vacancies through the matching function derived earlier. Hires decrease the stock of vacancies. Vacancies also decrease because some of the jobs for which vacancies were posted become unproductive. Vacancies increase as previously unproductive jobs, idle capacity, become potentially productive.

Using the two identities above, the dynamic system can be expressed solely in terms of U and V:

$$\begin{aligned} & dU(t)/dt = -h(t) + \pi_0(L-U(t)), \\ (4) \qquad & dV(t)/dt = -h(t) + \pi_1(K-L-V(t)+U(t)) - \pi_0 V(t), \\ & h(t) = V(t)(1-\exp[-aU(t)/V(t)]). \end{aligned}$$

Then, for given values of the two parameters π_0 and π_1, we can characterize the dynamics and steady state values of unemployment and vacancies, as well as of the flows of hires and separations.

It is convenient for later use to define the variable x by:

$$(5) \qquad x(t) = \pi_0(L-U(t))/V(t).$$

It is equal to the ratio of separations to vacancies being closed at any point in time. In steady state, separations are equal to hires so that x is equal to the proportion of closing vacancies which are filled; it can therefore be taken as an index of how tight or loose the labor market is. A smaller value of x is a tighter labor market. Note that, in steady state, there is a simple relation among x, U, and V, which will be useful below:

$$x = h/V = (1-\exp[-aU/V]),$$

or equivalently

$$V/U = -a/\log[1-x]. \tag{6}$$

Note that, at this point, we have given a full characterization of the behavior of unemployment and vacancies, and that it does not depend on the particular rule used by firms to choose among applicants. In our model, hiring rules do not affect how many are hired; but they determine who is hired, thus affecting the distribution of unemployment, as well as wages. In terms of conventional discussions of these issues, and of the framework developed in our previous papers ((1989), Olivier Blanchard (1989)) hiring rules do not affect the Beveridge curve but do affect the Phillips curve.

Section III. Ranking, unemployment duration and exit rates

We consider two hiring rules and derive their implications for the distributions of unemployment durations and exit rates. Under the first, firms do not rank applicants and choose randomly among them. Under the second, firms rank applicants, choosing the worker with the smallest unemployment duration first. We defer a discussion of the empirical evidence on hiring practices until the end of the section, once we have shown the implications of ranking rules.

We denote by $U(\theta,t)$ the pool at time t of those with unemployment duration less or equal to θ, so that $U(\theta,t)$ is the (unnormalized) distribution of unemployment. We denote its derivative with respect to duration, $dU(\theta,t)/d\theta$, by $u(\theta,t)$, so that $u(\theta,t)$ is the (unnormalized) density of unemployment with

duration equal to θ at time t. Finally, we denote by $e(\theta,t)$ the exit rate from unemployment at time t for those with duration equal to θ. When dealing with steady states, we simplify notation, and denote those variables by $U(\theta)$, $u(\theta)$ and $e(\theta)$ respectively.

1. Durations and exit rates under no-ranking

If, when they receive multiple applications, firms choose randomly among applicants, then all applicants have the same probability of being hired. Thus, the exit rate, the probability of being hired, is independent of duration and given by:

(7) $$e_0(t) = h(t)/U(t),$$

where we shall use the index 0 to denote variables when firms choose randomly among workers. $U_0(\theta,t)$ in turn satisfies:

(8) $$dU_0(\theta,t)/dt = -u_0(\theta,t) - e_0(t)U_0(\theta,t) + x(t)V(t),$$

where $x(t)$ was defined in (5). The change over time in the pool of unemployed of duration less or equal to θ is composed of three terms. The first two are outflows. The first is the flow of those whose duration now exceeds θ. The second is the flow of hires from the pool. The third term is an inflow, the flow of layoffs from employment.

In <u>steady state</u>, hires are equal to separations so that:

(9) $$e_0 = h/U = xV/U = -ax/\log[1-x],$$

using equation (6). The exit rate is a decreasing function of x. In steady state the distribution of unemployment durations is unchanging through time, so that, from (8):

$$u_0(\theta) = -e_0U_0(\theta) + xV.$$

Solving this differential equation in duration gives:

$$U_0(\theta)/U = 1 - \exp[-e_0\theta],$$

or using equation (9)

(10) $$U_0(\theta)/U = 1 - \exp[ax\theta/\log[1-x]].$$

2. Durations and exit rates under ranking

Suppose instead that, when firms receive multiple applications, they hire the worker with the shortest spell of unemployment. Why firms do so in our model is not specified. As all workers are identical, any rule is, for a given firm, as good as any other, and no individual firm has an incentive to change its hiring rule. An alternative assumption, with equivalent implications, would be that there is an arbitrarily small deterioration of skills with unemployment duration, so that, while all workers are acceptable, the firm marginally prefers those who have been unemployed the least time[7]. In a steady state, a similar argument can be made from the presence of a small number of unemployables, who are a larger fraction of the unemployed of longer duration. Whether firms actually perceive large differences between unemployed workers of different durations is a separate, empirical, issue that we discuss below.

In the case of ranking, the equation characterizing $U(\theta,t)$ is given by:

(11) $$dU(\theta,t)/dt = -u(\theta,t) - V(t)(1-\exp[-aU(\theta,t)/V(t)]) + x(t)V(t).$$

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[7] This argument implicitly assumes that all workers have to be paid the same wage. But this is an implication of our assumptions about wage determination below.

The change in the pool of unemployed of duration less or equal to θ at time t is again composed of three terms. The first and third are the same as before. The first is the flow of those whose durations now exceeds θ. The third is the flow of layoffs. The second term is the flow of those who find a job and leave the pool and is derived as follows. Consider a given vacancy. This vacancy will result in a hire from the pool $U(\theta,t)$ if there is at least one application from a worker from that pool. The probability that there is at least one application from a worker in the pool $U(\theta,t)$ is equal to one minus the probability that there is no application from that pool, thus equal to $(1-\exp[-aU(\theta,t)/V(t)])$. If $V(t)$ is the flow of vacancies being closed at any point, the flow of hires from $U(\theta,t)$ is thus equal to $V(t)(1-\exp[-aU(\theta,t)/V(t)])$.

The exit rate for a worker of duration θ at time t is equal to the probability that the worker applies, a, times the probability that the vacancy applied for has no application from an unemployed worker with duration less than θ. Thus:

(12) $$e(\theta,t) = a \exp[-aU(\theta,t)/V(t)].$$

In <u>steady state</u>, the pool of unemployed with duration less than or equal to θ is constant, so that, from equation (11):

(13) $$u(\theta) = - V(1-\exp[-aU(\theta)/V]) + xV.$$

Solving this differential equation in duration gives:

$$x \exp[a(x-1)\theta] = (x-1)\exp[aU(\theta)/V] + 1.$$

Solving for $U(\theta)$ and using equation (6) gives the distribution of durations as a function of x:

(14) $$U(\theta)/U = 1 - \{\log[1-x.\exp[-a(1-x)\theta]]/\log[1-x]\}.$$

The exit rate is given in turn by:

$$e(\theta) = a \exp[-aU(\theta)/V].$$

Using (14) and (6) gives the exit rate as a function of x:

(15) $$e(\theta) = a(1-x)/(1-x.\exp[-a(1-x)\theta]).$$

What are the implications of equation (15) for the behavior of exit rates? First, and in contrast to the case of no-ranking, the exit rate is a decreasing function of duration, θ. For $\theta=0$, the exit rate is equal to a: a worker who has just become unemployed and applies to a vacancy is sure to be first and thus to be hired. As θ goes to infinity, the exit rate converges to $a(1-x)$.

Second, the exit rate is a decreasing function of x, the state of the labor market. From equation (15) and using l'Hospital's rule, as x tends to 1, that is as the labor market becomes more depressed, the distribution of exit rates tends to the limiting distribution:

$$e(\theta) \rightarrow a/(1+a\theta) \quad \text{as } x \rightarrow 1.$$

Third, the effect of unemployment duration on the exit rate is stronger the more depressed the labor market. More precisely, $\delta^2\log[e(\theta)]/\delta\theta\delta x$ is negative[8]. The intuition for this result is simple. For low values of x, the ratio of applications to vacancies is low: most vacancies receive zero or one application and the long term unemployed stand nearly as good a chance of being hired as the short term unemployed. The exit rate therefore decreases slowly with θ. As x increases, the ratio of applications to vacancies increases, and with it the

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8 This is shown as follows: $\delta^2\log(e(\theta))/\delta\theta\delta x$ has the same sign as $\exp[a(1-x)\theta]\{2x-1-a\theta x(1-x)\}-x^2$. That expression is negative for x=0 and x=1, and does not change sign between 0 and 1.

likelihood of multiple applications. The long term unemployed are then much less likely to be hired than the short term unemployed[9].

Figure 1 plots $U(\theta)/U$ and figure 2 plots $e(\theta)$ for two values of x, $x=0.1$ and $x=0.9$. These extreme values show clearly the effects of ranking on exit rates and duration distributions. In both a is taken to be 0.1. The value $x=0.1$ corresponds to a tight labor market. From equation (6), the ratio of applications aU to vacancies V is equal to 0.11. Most application windows are closed without being filled, and very few generate multiple applications; the proportion of vacancies which receive multiple applications is equal to 0.5%. The probability of getting a job is high, no matter what length of unemployment spell a worker has gone through. The exit rate is therefore high and declines very slowly with unemployment duration.

The value $x=0.9$ corresponds to a depressed labor market, with a ratio of applications to vacancies of 2.3. 90% of vacancies are filled when application windows close, and 67% generate multiple applications. Thus the exit rate declines rapidly with duration.

3. Evidence on hiring rules.

Having shown the implications of ranking rules, we now briefly turn to the

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9 This implication may not hold in a model in which workers have quality differences. The decline in the exit rate then reflects the deteriorating composition of the pool, and the effect may be less drastic when unemployment is high than when it is low. Put another way, being long term unemployed may be a weaker correlate of bad quality when there are many long term unemployed.

empirical evidence on hiring practices. The evidence comes from a number of case studies, in particular a 1982 study for the US reported by John Barron and John Bishop (1985), and a 1986 study for the UK by Nigel Meager and Hilary Metcalf (1987)[10].

Our assumption that ranking by duration is important is supported by three sets of facts from those studies. First, many vacancies generate a large number of applications, a necessary condition for our model to be of empirical relevance (studies cited above, and Harry Holzer, Lawrence Katz and Alan Krueger (1988)). Second, in many cases, the position is filled without interviewing all applicants but by first creating a short list based on some simple criterion. Third, "time since last job" is used both as a short listing criterion, and as an important criterion in a final decision. Supportive evidence also comes from the perceptions of the employed workers: in the UK in 1985, while unemployment stood at 11.6%, the proportion of employed workers who thought they could find a job quickly if laid off stood at 45%, slightly higher than the 40% giving the same answer in 1977 when unemployment stood at only 5.2% (Olivier Blanchard and Lawrence Summers (1988)).

However our specific assumption is stronger than simply saying that ranking by unemployment duration is important. We assume that unemployment duration is used as the only criterion in hiring. The empirical evidence shows clearly that

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10 This last study is particularly interesting given the depressed state of the labor market in the UK and the high proportion of long term unemployed at that time.

this assumption is too strong. Leaving aside the fact that firms clearly take into account the personal characteristics of workers - an aspect we have ignored by assuming identical workers - unemployment duration is not the only way in which firms look at the employment history of workers. A criterion mentioned more often than "time since last job" is "employment record". What this means exactly is unclear; it may well be that a recent but short period of employment may be discounted by firms. If the length of previous employment as well as the duration of unemployment matter, the analysis becomes substantially more complicated. More importantly, our qualitative results about wage determination may be substantially affected. We shall return to this issue in the conclusion.

We also assume that, while firms use ranking by unemployment duration, they see all workers as equivalent. As we indicated, the assumption can be relaxed to allow for some duration dependence of skills, so long as all workers are considered acceptable by firms. The evidence is that firms which rank by unemployment duration perceive the long term unemployed as distinctly worse, often as lacking motivation. Whether those firms would hire the long term unemployed were they the only workers available is unclear from the available evidence.

Section IV. Wage determination under Nash bargaining

In characterizing wage determination, we make two assumptions. The first is Nash bargaining between the newly hired worker and the firm. The second is that workers cannot sign binding contracts[11]. This implies that, once an unemployed -possibly a long term unemployed- worker is hired, she can renegotiate with the firm, but now as an employed worker. This allows us to assume that the wage is determined by Nash bargaining between the firm and each employed worker.

To see the importance of the second assumption, suppose instead that workers signed binding contracts. Then, the surplus to a worker from a match would depend on the length of her unemployment spell, and so would the wage under Nash bargaining. Moreover, firms would prefer workers ready to accept lower wages, thus upsetting any preferred hiring pattern over identical workers. The presence of multiple applications would raise additional issues. When a firm had multiple applications, and if the applicants could sign binding contracts, the presence of two or more competing workers would yield the Bertrand rather

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[11] This issue arises also in efficiency wage models, and in insider/outsider models, where it has been discussed at length. We have little to add to the debate. We return to the issues raised by bonding and commitment in the conclusion.

than the Nash solution[12].

We first characterize the wage under Nash bargaining and random choice of workers by firms in hiring; this yields a familiar formula. We then characterize the wage under Nash bargaining and ranking.

1. Wages under no-ranking.

Under random choice of applicants, the Nash bargaining solution takes a form familiar from the earlier literature on search (e.g., Peter Diamond (1982)). Let $W_e(t)$ and $W_u(t)$ be the values of being employed and unemployed respectively. Let r, w(t), π_0 and e(t) be the interest, wage, separation and exit rates. Under no-ranking, the exit rate is the same for all unemployed workers and is simply equal to $e_0(t) = h(t)/U(t)$. Assume that there is no flow of benefits when unemployed. $W_e(t)$ and $W_u(t)$ satisfy the two arbitrage equations:

(16) $r\, W_e(t) = w(t) + \pi_0\,(W_u(t)-W_e(t)) + dW_e(t)/dt,$

(17) $r\, W_u(t) = e_0(t)(W_e(t)-W_u(t)) + dW_u(t)/dt.$

Symmetrically, let W_f, W_v, and W_i denote the values of a filled job, of a vacant job, and of an idle job. Those values satisfy the three arbitrage equations:

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12 The assumption that the firm first chooses a worker among the applicants and then bargains with that worker would give us Nash rather than Bertrand bargaining, but, with binding contracts, the outcome would still depend on the unemployment spell of the selected worker. We have not explored whether the assumption that newly hired workers must be paid the same wage as the currently employed can deliver results similar to those we derive under individual bargaining.

$$(18)\qquad r\,W_f(t) = y-w(t) \qquad\qquad +\pi_0(W_i(t)-W_f(t))+ dW_f(t)/dt,$$

$$(19)\qquad r\,W_v(t) = (h(t)/V(t))(W_f(t)-W_v(t))+\pi_0(W_i(t)-W_v(t))+ dW_v(t)/dt,$$

$$(20)\qquad r\,W_i(t) = \qquad\qquad \pi_1(W_v(t)-W_i(t))+ dW_i(t)/dt.$$

Productive and filled jobs bring a revenue to the firm of y-w(t). They may however, with probability π_0, become unproductive, and thus idle. Vacant jobs do not bring revenue and may either become filled, with probability h/V, or unproductive and idle with probability π_0. Idle jobs in turn may become potentially productive and thus vacant with probability π_1.

Under Nash bargaining, the surplus from a match is split equally between the worker and the firm so that:

$$(21)\qquad W_e(t) - W_u(t) = W_f(t) - W_v(t).$$

We defer the examination of the dynamics to the next section and concentrate on the steady state. In <u>steady state</u> all W's are constant, and the wage is given by:

$$(22)\qquad w/y = (r+\pi_0+e_0)/(2r+2\pi_0+e_0+(h/V))$$

$$= (r+\pi_0+(h/U))/(2r+2\pi_0+(h/U)+(h/V)).$$

Using the fact that in steady state h/V=x and h/U = -axlog[1-x] (from equation (6)) gives the wage as a function of x, r and π_0. Finally, if we assume that r and π_0 are small in relation to both e_0 and h/V, as is empirically the case, then the wage reduces to:

$$w/y \approx V/(V+U).$$

This gives the wage as a simple function of unemployment and vacancies.

2. Wages under ranking

How do things change if firms rank by unemployment duration, so that exit

rates from unemployment depend on duration? The value of being unemployed now depends on duration. Thus let $W_u(\theta,t)$ be the value of being unemployed with duration θ; and $W_e(t)$, the value of being employed. The arbitrage equations characterizing the behavior of $W_u(\theta,t)$ and $W_e(t)$ are then given by:

(23) $$r\, W_e(t) = w(t) + \pi_0(t)(W_u(0,t)-W_e(t)) + dW_e(t)/dt,$$

(24) $$r\, W_u(\theta,t) = e(\theta,t)(W_e(t)-W_u(\theta,t)) + dW_u(\theta,t)/d\theta + dW_u(\theta,t)/dt,\ \forall\theta.$$

When an employed worker becomes unemployed, her unemployment duration is zero, so that the change in value from becoming unemployed is now $W_e(t)-W_u(0,t)$. An unemployed worker of duration θ either finds a job with probability $e(\theta,t)dt$, or becomes unemployed with duration $\theta+d\theta$.

The arbitrage equations characterizing the values of filled, vacant and idle jobs are the same as before.

As we argued earlier, in the absence of binding contracts, we can think of the wage as the result of Nash bargaining between an employed worker and a firm. If a deal is not struck, an employed worker stands to lose $W_e(t)-W_u(0,t)$ as she becomes an unemployed worker with zero length of unemployment spell; the firm stands to lose $W_f(t)-W_v(t)$. The Nash bargain is characterized by:

(25) $$W_e(t) - W_u(0,t) = W_f(t) - W_v(t).$$

In the rest of this section, we concentrate on the steady state. In <u>steady state</u>, equation (24) becomes:

$$r\, W_u(\theta) = e(\theta)(W_e-W_u(\theta)) + dW_u(\theta)/d\theta.$$

Integrating forward with respect to duration gives[13]:

$$W_u(0) = W_e \int_0^\infty e(\theta)\ (\exp[-\int_0^\theta (e(z)+r]dz)\ d\theta.$$

Using this expression for $W_u(0)$ in (25), and solving for the wage gives:

(26) $$w/y = (r+\pi_0+e^*)/(2r+2\pi_0+e^*+(h/V)),$$

with e^* implicitly defined by:

(27) $$e^*/(e^*+r) = \int_0^\infty e(\theta)\ \exp[-\int_0^\theta (e(z)+r)dz]\ d\theta.$$

The formula characterizing the wage is thus the same as under random choice, except for the presence of e^* rather than e_0. This however is an essential difference and we explore it at more length.

How does the labor market situation now affect the wage? From the firm's side, labor market tightness affects the wage through (h/V), which reflects how long the firm expects to have to wait for another applicant, were the deal with the current worker not to go through. From the worker's side, market tightness affects the wage through the distribution of exit rates from unemployment. In considering the threat point associated with an end to current employment, a worker must consider reemployment possibilities in the future. Thus, what matters for a worker is not the current unemployment rate, but the sequence of exit rates she would face if she became unemployed. The effect of that sequence

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[13] The limit of $W_u(\theta)$ as θ grows without limit is found by noting that $W_u(\theta)$ is monotonically decreasing since $e(\theta)$ is monotonically decreasing. Thus setting the derivative equal to zero and substituting the limit value of $e(\theta)$ gives the limit value of $W_u(\theta)$ in terms of W_e.

is summarized by e^*. $e^*/(e^*+r)$ is a weighted sum of exit rates for durations 0 to ∞, with the weights depending on the exit rate itself. If exit rates did not decline with duration, e^* would be equal to e_0, the duration independent exit rate with random hiring, and the expression for the wage would reduce to that above.

The expression for $e^*/(e^*+r)$ can be rewritten in terms of unemployment duration densities, showing more clearly the effects of the composition of the unemployment pool in steady state. Differentiating equation (13), we have:

$$u'(\theta) = -a\exp[-aU(\theta)/V]u(\theta) = -e(\theta)u(\theta).$$

Integrating with respect to duration, we have:

$$u(\theta) = u(0)\exp[-\int_0^\theta e(z)dz].$$

Using this expression in equation (27) gives:

$$e^*/(e^*+r) = \int_0^\infty e(\theta)(u(\theta)/u(0))\exp[-r\theta]d\theta = \int_0^\infty -u'(\theta)\exp[-r\theta]/u(0)d\theta.$$

Integrating by parts, gives e^* as a function of the sequence of $u(\theta)$:

$$e^*/(e^*+r) = 1 - r\int_0^\infty u(\theta)\exp[-r\theta]/u(0)d\theta. \tag{28}$$

This expression shows how a change in the pattern of exit rates, with layoffs and total unemployment unchanged, that shifted the distribution of durations toward greater durations raises $e^*/(e^*+r)$, and thus e^* and w. Since the exit rate with ranking is monotonically decreasing, crossing the no-ranking exit rate once, this implies that the wage is higher with ranking than with no-ranking.

Finally, using the distribution of exit rates given in equation (15), e^* can be expressed as a function of x:

$$\text{(29)} \qquad e^*/(e^*+r) = (1-x)\, x^{-1-r/(a(1-x))}\, B_x\left((r+a(1-x))/a(1-x), 1\right),$$

where B_x is the incomplete Beta function,

$$B_x(p,q) = \int_0^x \tau^{p-1} (1-\tau)^{q-1}\, d\tau.$$

While not particularly revealing, this expression allows us to characterize the effects of x on e* and thus on w.

Compared to random choice, how large a difference does ranking actually imply for the steady state level of the wage at any level of unemployment, and for the effect of changes in steady state unemployment on the wage? The answer depends greatly on the interest rate. A positive value of r is, in steady state, the only reason why the perspective of the employed workers differs from that of those already unemployed. For conventional values of r, the difference between the ranking and no-ranking wages is very small. This is shown in figure 3a, which gives the ranking and no-ranking wages as a function of x, assuming that the probability of an application is .1 per week, y is 1.0 and the annual interest rate is 10% (the weekly interest rate is .2%). Even as x becomes close to 1, as the labor market is more depressed, the ranking and no-ranking wages remain close.

One may however argue that the relevant interest rate is higher, at least for the workers if not for the firm[14]. Figure 3b shows the effect of x on the wage for a value of the weekly interest rate of 1%. The effects of ranking on

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[14] This argument however suggests using different interest rates for the firm and for workers, something we have not done in the derivation.

both the level of the ranking wage and its unemployment elasticity become clearer. The difference between the ranking and no-ranking wages increases as x increases. The elasticity of the ranking wage with respect to unemployment steadily decreases as x becomes close to one, as the labor market becomes more depressed. But it is clear from those figures that large steady state effects from ranking alone require large discount rates, larger than we are willing to assume.

Intuition suggests that the effects of ranking on the relation between unemployment and wages may be more dramatic out of steady state. For example, in a labor market which is depressed but picking up, the prospects of the currently employed, were they to become unemployed may be very different from the current experience of those currently unemployed, leading to substantial pressure on wages. Two effects are at work here. The first would be present even under random hiring: higher hiring rates improve prospects for all unemployed workers. But in addition, ranking improves prospects more for those with short unemployment duration. The next section focuses on dynamics.

Section V. Dynamics of exit rates, durations and wages

Dynamics of aggregate unemployment, vacancies, separations, and hires are characterized by equation (4), those of exit rates and durations by (7) and (8) for the no-ranking case, and by (11) and (12) in the case of ranking. The equations characterizing wage behavior are equations (16) to (21) for the case of no-ranking, equations (18) to (20), and (23) to (25) for the case of ranking. However, characterizing dynamics in the case of ranking is too hard an

analytical task, and we turn now to simulations. Simulations are carried out using a discrete time model; the details are given in Appendix A. We simply note here that the simulation model is the discrete time version of the continuous time model above except for the treatment of vacancies. The unit period is a week. It is simpler to return to a specification with non overlapping vacancies, as in the first part of section I, with all application windows being open for one week at a time.

The parameters in the model are a, the probability that an unemployed worker makes an acceptable application during a week, π_0 and π_1 which characterize the process of job creation and destruction, the labor force, L, the capital stock, K, and the interest rate, r. We choose parameters so as to - very roughly- replicate the characteristics of the US labor market. We shall focus on movements between two steady states, one which corresponds to a depressed labor market, the other to a normal labor market.

We choose, as a normalization, the labor force, L, to be 1. We choose K to be 1.05. We choose the interest rate equal to .1% per week[15]. We choose a to be 0.7: unemployed workers make 0.7 acceptable applications per week.

To choose the π parameters, we note -as in our previous papers- that, given π_0 and π_1, the proportion of productive jobs (filled or vacant) is equal to $c = \pi_1/(\pi_0+\pi_1)$ and that the proportion of jobs which go from being productive to being unproductive is equal to $s = \pi_0\pi_1/(\pi_0+\pi_1)$. c can be thought of as an

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[15] As is clear from our steady state results in the previous section, the choice of such an interest rate implies nearly no difference between steady state ranking and no ranking wages.

index of aggregate activity, s as an index of the intensity of reallocation. As we are interested in cyclical fluctuations, we consider steady states which differ in their value of c, but have the same value of s.

The first set of parameters, corresponding to a depressed labor market, is $\pi_0 = .019$ and $\pi_1 = .129$, which in turn imply c=.872 and s=.0165. Together with the values of a,L and K given above, these parameters imply steady state values for unemployment of .102, for vacancies of .018, and for x, the index of labor market tightness we focused on earlier, of .94.

The second set of parameters, corresponding to a "normal" labor market, is $\pi_0 = .018$ and $\pi_1 = .22$, which imply c=.925 and s=.0165. Together with the values of a,L and K, these parameters imply steady state values for unemployment of .05, for vacancies of .021, and for x of .80.

1. A sharp decrease in unemployment.

The first simulation shows the effects of a sharp decrease in unemployment starting from a depressed labor market. In that simulation, the index of aggregate activity, c, increases unexpectedly and permanently from .872 to .925, with expectations adjusting at once to the permanent change. This leads to a decrease in unemployment from an initial value of 10.2% to a new steady state value of 5.0%. The speed at which unemployment decreases exceeds that found in actual economies; the reason for presenting this simulation is that the sharp and rapid decline in unemployment shows most clearly the mechanisms at work.

Figure 4.a presents the path of adjustment of unemployment and vacancies. Time is measured in weeks on the horizontal axis. Unemployment adjusts

monotonically to its lower value, with the adjustment being largely over after 4 months. Vacancies increase from 1.8% to 2.1%, overshooting their steady state value for some time. This is because the number of job creations, thus of newly posted vacancies is larger at the beginning, when activity picks up, than when the economy nears steady state.

Figure 4.b gives unemployment densities by duration at various points in the adjustment process. The final steady state distribution shows lower density at all durations than the initial distribution, reflecting the tighter labor market situation. Note that, during the process of adjustment, densities at low duration decrease to values lower than their steady state values. This is because the large initial increase in job creations and thus newly posted vacancies leads to large hiring rates, and thus to rapid attrition of those cohorts entering unemployment after the pickup in economic activity and hiring. This is confirmed in figure 4.c which gives exit rates by duration, again at various points in the adjustment process. In the final steady state, exit rates are much higher than in the initial steady state, especially at long durations. And exit rates, 4 and 8 weeks after the change in c, substantially exceed their steady state values.

Finally, figure 4.d gives the behavior of wages. Both the ranking and the no-ranking wages increase substantially in anticipation of higher exit rates in the future. The effect on the ranking wage is substantially stronger. It more than doubles, overshooting its steady state value and then slowly decreasing over time. This shows most clearly the effects of ranking. Although the market is still initially depressed, the prospects of the employed workers are so good as to increase the wage above its steady state value.

2. A slower decline in unemployment.

The second simulation considers the same decrease in unemployment, but at a slower pace. In that simulation, c follows the partial adjustment process $c(t)-c(t-1) = .02(.926-c(t-1))$, starting from an initial value of .872. The change from the initial value is unanticipated, but from then on the path of adjustment of c is fully anticipated by workers and firms.

Figure 5.a shows the adjustment of unemployment and vacancies over the first two years. The process of adjustment comes now from the convolution of the process for c and the intrinsic dynamics of the system. The process of adjustment is monotonic for both vacancies and unemployment, and is mostly complete after two years.

Figures 5.b and 5.c show the evolution of unemployment densities and exit rates. There is no longer overshooting of either exit rates or densities.

Figure 5.d shows the behavior of wages. Despite the fact that the adjustment of unemployment is now much slower, the contrast between the ranking and the no-ranking wages is still dramatic. While the ranking wage no longer overshoots its steady state value, it increases by a large amount when the labor market picks up, substantially more than the no-ranking wage.

Section VI. Efficiency Wages

We have analyzed the labor market under the assumption that the wage is set to divide the gain from beginning employment between a firm and a newly laid-off worker,

$$W_e - W_u(0) = W_f - W_v.$$

An alternative assumption is that wages are set by firms, rather than bargained over. The determinants of the optimal wage include the need to attract workers more quickly, to attract better workers, to hold workers, and to encourage workers to work more efficiently. One particularly simple version of this array of effects is the case where wages are set to just satisfy an equality between the cost of losing a job and a constant. This condition has been interpreted as a no-shirking condition (Guillermo Calvo (1979), Carl Shapiro and Joseph Stiglitz(1984)). If a fired worker is in the same position as a newly laid-off worker, this condition becomes:

(30) $$W_e - W_u(0) = \Omega.$$

This condition can only hold where the gain to hiring a worker remains positive. When the condition holds, no one is ever fired, and the dynamics of unemployment (and, we assume, vacancies) is the same as that modeled above. Thus, the presence of duration dependent (rather than random) hiring rules affects the wage. The equations for the values of different positions of workers in terms of the wage are unchanged. Using (30) to determine the wage and solving, we have:

(31) $$w = (r+\pi_0+e^*)\Omega.$$

Similarly, it is straightforward to simulate the economy, using this alternative wage determination equation and following the same discrete time formulation as was used above and is described in Appendix A. Figures 6 and 7 show the response to the same sudden and slow increases in c which were discussed above. For these simulations the parameter Ω was chosen so that the wage with ranking would be the same before the change with efficiency and Nash

bargained wages. These figures give the same qualitative picture as those above.

It is natural to ask what happens if we combine Nash and efficiency arguments about the determination of wages. That is, how are wages determined if the firm and the worker are free to bargain over the wage, but it is known that the worker will shirk if the wage is below the level needed for the no-shirking condition. As we argue in Appendix B, the wage is the maximum of the wages set by the two conditions. Thus, the no-shirking condition results in the higher of two wage levels, just as is the case with an outside option (Kenneth Binmore (1983) and Avner Shaked and John Sutton (1984)).

Section VII. Conclusions

Rather than repeat our results, we take up one implication of our results for microeconomic work on unemployment, and discuss two issues in our basic model.

It is a common practice to make comparative static inferences from observed exit rates of the unemployed, using the exit rates as contributions to hiring. The model of this paper points to the dangers of doing so. (On the same issue, in a different model, see Bo Axell and Harald Lang (1988).) Consider first the random hiring model. In this model, all workers have the same exit rates. Yet this exit rate is not equal to the marginal contribution of a worker to aggregate hiring. With a constant returns aggregate matching function, the marginal product of a worker will be less than her average product. Consider next the ranking model. In this model all workers have exactly the same

marginal contributions to aggregate hiring, but workers differ significantly in their exit rates. Thus two economies with the same numbers of vacancies and unemployed but different ranking rules, and so different distributions of durations, would have the same aggregate hires. Consider finally an extension of the ranking model where the acceptable application rate, a, varies with duration. For example, assume that long duration unemployed, knowing they have little chance of finding jobs are more diligent in finding vacancies to which to apply. (Empirical evidence points to the opposite behavior, but we are not making an empirical point here.) If the application rate rises sufficiently slowly with duration, the exit rate could decline with duration while the marginal contribution to aggregate hires is rising with duration. These examples underline the importance of an articulated equilibrium model when considering the effects of policies such as changes in the schedule of unemployment benefits.

Our results are based on the assumptions that there is no deterioration of skills with unemployment duration and that workers cannot sign binding contracts. How are the results likely to be modified when we relax one or both assumptions? If we allow for - even partial - commitment but maintain the assumption of identical workers, the only equilibrium is one with random hiring. The reason is simple: if other firms rank by unemployment duration, an individual firm has an incentive to hire those who have been unemployed for the longest time, as they will accept a lower compensation. Ranking cannot therefore be an equilibrium. If instead, we do not allow for commitment, but allow for duration dependence, so that, for example, the training cost of a new worker increases with unemployment duration, then, our model still applies, with

the modification that those with very long unemployment duration may require too large a training cost and so may no longer be employable. In contrast to the version in the text however, firms are no longer indifferent with respect to the hiring rule, but now have an incentive to rank applicants and choose those with the shortest duration first. This implies that, if we allow for both commitment and duration dependence, the ranking rule we have used will still be an equilibrium when the training cost minus the bond that firms extract from workers in equilibrium increases with duration. In a companion paper, (1990b), we are exploring equilibrium with training costs and commitment.

We also speculate that similar considerations are at work if - as empirical evidence suggests is the case - firms care not only about unemployment duration, but also about the employment record. Suppose for example that firms rank workers by unemployment duration, subject however to the constraint that employment duration in the previous job exceeds some minimum length. In this case, it is clear that firms will be able in effect to extract a bond from the worker, with the size of the bond being a function of the length of employment required to acquire a badge of good behavior. A question is then when, with duration dependence, ranking is still an equilibrium.

Appendix A

The model used for simulations is a discrete time version of the model in the text, except for the treatment of the timing of vacancies. It is simpler in discrete time to shift back to a model without overlapping application windows.

Our basic period for the simulations is a week. We start the week with E(t-1) filled jobs, U(t-1) unemployed, V(t-1) vacancies, and K-V(t-1)-E(t-1) in idle capacity. From Monday to Friday, the employed workers produce, the unemployed make job applications, and vacancies receive job applications. On Saturday, there are π_0 and π_1 shocks that change the productivity of some of the jobs. On Sunday, some workers are laid off while others get new jobs. On Monday, the wage is agreed to for the coming week and work begins. The aggregate shocks we consider in simulations are changes in the values or the paths of π_0 and π_1. These changes occur on Saturday.

By normalization of the labor force,

(A1) $$E(t) + U(t) = 1.$$

The hiring function is equation (1) in the text. Vacancies shrink from new hires and from idling while they grow from positive shocks to idle capacity:

(A2) $$V(t) = V(t-1)*\exp[-a*U(t-1)/V(t-1)]*(1-\pi_0) + \pi_1*(K-E(t-1)-V(t-1)).$$

Implicit in this formulation is the assumption that newly hired workers who are idled at the end of the week do not get another crack at a job this week.

The newly unemployed, U(0,t), were laid off at the end of week t-1. Their numbers satisfy:

(A3) $$U(0,t) = \pi_0 E(t-1).$$

We denote by $U(t,\theta)$ the numbers of unemployed who were laid off less than $\theta+1$ full weeks ago. We track θ on a weekly basis from 0 to 51 and denote total unemployed by $U(t)$. We assume that the ranking model holds for durations up to one year, but all unemployed with durations over a year have the same job finding probabilities. We need to track the unemployment duration equations for the weeks up to one year and for total unemployment. For $\theta = 1$ to 52, the unemployed up to any duration are the unemployed up to a duration one week less one week ago, plus the newly laid off minus the new hires that survive the bad productivity shock within the week:

(A4) for $\theta=1$ to 51,

$$U(\theta,t) = U(\theta-1,t-1) + U(0,t) - V(t-1)*(1-\exp[-a*U(\theta-1,t-1)/V(t-1)])*(1-\pi_0);$$

(A5)

$$U(t) = U(t-1) + U(0,t) - V(t-1)*(1-\exp[-a*U(t-1)/V(t-1)])*(1-\pi_0).$$

We now have a recursive system which will track vacancies and unemployment of different durations. Associated with those values of unemployment and vacancies are the exit rates (under ranking):

(A6)

$$e_0(0,t+1)= V(t)*(1-\exp[-aU(0,t)/V(t)])*(1-\pi_0)/U(0,t);$$

and for $\theta=1$ to 52,

$$e(\theta,t+1)= V(t)*(\exp[-aU(\theta-1,t)/V(t)] -\exp[-aU(\theta,t)/V(t)])*(1-\pi_0)/(U(\theta,t)-U(\theta-1,t)).$$

It is convenient to define $f(t+1)$ as $h(t+1)/V(t)$:

$$f(t+1)=(1-\exp[-aU(t)/V(t)])*(1-\pi_0).$$

Next, we derive the equations for the Nash bargained wage. The value functions for the different positions for a firm satisfy:

$$W_f(t-1) = y-w(t-1) + \delta(\pi_0 W_i(t)+(1-\pi_0)W_f(t));$$

(A7)

$$W_v(t-1) = \delta(\pi_0 W_i(t)+f(t)W_f(t)+(1-\pi_0-f(t))W_v(t));$$

$$W_i(t-1) = \delta\{(1-\pi_1)W_i(t) + \pi_1 W_v(t)\}.$$

Define the value from filling a job as Δ_f:

(A8) $$\Delta_f(t) = W_f(t) - W_v(t).$$

Subtracting the first two equations in (A7), we have:

(A9) $$\Delta_f(t-1) = y-w(t-1) + \delta(1-\pi_0-f(t))\Delta_f(t).$$

The value functions for the workers recognize the constant probability of job finding for all workers with duration over one year.

(A10) $$W_e(t-1) = w(t-1) + \delta\{p_0 W_0(t)+(1-p_0)W_e(t)\};$$

and for $\theta = 0$ to 51,

$$W_u(\theta,t-1) = \delta\{e(\theta,t)\ W_e(t) + (1-e(\theta,t))W_u(\theta+1,t)\};$$

$$W_u(52,t-1) = \delta\{e(52,t)W_e(t) + (1-e(52,t))W_u(52,t)\}.$$

Similarly, we define the gain from finding a job after different unemployment durations:

(A11) for $\theta = 0$ to 52, $\Delta(\theta,t) = W_e(t) - W_u(\theta,t)$.

Subtracting the equations in (A10) and using the gains, (A11), we have:

(A12) for $\theta = 0$ to 51,

$$\Delta(\theta,t-1) = w(t-1) + \delta\{(1-e(\theta,t))\ \Delta(\theta+1,t)-\pi_0\Delta(0,t)\};$$

$$\Delta(52,t-1)= w(t-1) + \delta\{(1-e(52,t))\Delta(52,t)\ -\pi_0\Delta(0,t)\}.$$

Subtracting the 0 equation from the rest, we can write:

(A13) for $\theta = 1$ to 51,

$$\Delta(\theta,t-1) =\Delta(t-1,0)+\delta\{(1-e(\theta,t))\Delta(\theta+1,t)-(1-e(0,t))\Delta(t,1)\};$$

$$\Delta(52,t-1)=\Delta(t-1,0)+\delta\{(1-e(52,t))\Delta(52,t)-(1-e(0,t))\Delta(t,1)\}.$$

The generalized Nash bargaining solution relates the loss from becoming newly unemployed to the loss from becoming a vacancy:

(A14) $$\Delta(0,t)=W_e(t)-W_u(0,t)=z\{W_f(t)-W_v(t)\} =z\Delta_f(t).$$

(We have implicitly taken z to be 1 in the text. We do the same in the simulations we report in section 5.) Combining this with the equations for the gains we have:

(A15) $\Delta(0,t-1)+\Delta_f(t-1) = (1+1/z)\Delta(0,t-1) = (1+z)\Delta_f(t-1)$

$$= y+\delta\{(1-\pi_0-f(t))\Delta_f(t)+(1-e(0,t))\Delta(1,t)-\pi_0\Delta(0,t)\}.$$

Thus, we have a set of 54 forward looking value equations. Starting at the steady state value equations, the equations can be run backwards using the escape rates along the transition path and starting far enough in the future that the value equations are indistinguishable from the final steady state.

From this trajectory of values of Δ_i, the wage can be calculated. In particular, we use the equation:

(A16) $(1+z)w(t-1)=zy+\delta\{(1-f(t))\Delta(0,t)-(1-e(0,t))\Delta(1,t)\}.$

To calculate the steady state values for starting this calculation, we use the equations:

(A17) $\delta(1-e(0))\Delta(1) = - y + \Delta(0)\{(1+1/z)(1+\delta\pi_0)-\delta(1-f)/z\};$

(A18) for $\theta = 1$ to 52, $\Delta(\theta) = \alpha(\theta)\{\Delta(0)-\delta(1-e(0))\Delta(1)\}$,

where $\alpha(\theta)$ satisfies $\alpha(52) = 1/\{1-\delta(1-e(52))\}$,

and for $\theta = 1$ to 51, $\alpha(\theta) = 1 + \delta(1-e(\theta))\alpha(\theta+1)$.

The first two equations solve simply for Δ_0 and Δ_1 and can be used to solve the remaining steady state values.

For comparison purposes, we also want to calculate the wage assuming <u>random hiring</u>. The trajectories of U and V are unaffected. e(t+1) and f(t+1) now satisfy:

(A19) $e(t+1) = V(t)*\{1-\exp[-aU(t)/V(t)]\}*(1-\pi_0)/U(t);$

$f(t+1) = \{1-\exp[-aU(t)/V(t)]\}*(1-\pi_0).$

The value equations for the firm continue to satisfy (A9). For the worker, we have:

(A20) $$W_e(t-1) = w(t-1)+\delta\{\pi_0 W_u(t)+(1-\pi_0)W_e(t)\};$$

$$W_u(t-1) = \delta\{e(t)W_e(t)+(1-e(t))W_u(t)\}.$$

Subtracting,

(A21) $$\Delta_e(t-1) = W_e(t-1) - W_u(t-1)$$

$$= w(t-1) + \delta(1-\pi_0-e(t))\Delta_e(t).$$

Since the Nash bargaining solution, (A14), still holds, we have:

(A22) $$\Delta_e(t-1)+\Delta_f(t-1) = (1+1/z)\Delta_e(t-1) = (1+z)\Delta_f(t-1)$$

$$= y + \delta\{(1-\pi_0-f(t))\Delta_f(t)+(1-\pi_0-e(t))\Delta_e(t)\}.$$

As before, the simulation is done by calculating values backwards from a steady state.

We also derive the wage under <u>efficiency wages</u>. To simulate efficiency wages, we continue to use (A13). Instead of (A15), we use the no-shirking condition:

(A23) $$\Delta(0,t) = \Omega.$$

Thus we have 52 forward looking equations which can be solved backwards from the steady state. The wage can then be solved from any equation in (A12) and the nonnegativity of the value of hiring a worker checked from solving (A9) backwards. To calculate the steady state values, we use (A23) along with (A18), which continues to hold. For the firm, we use the steady state version of (A9).

Appendix B: Combining Nash and Efficiency Wage Theories

In the text we have considered separately the situations where the wage satisfies the Nash bargaining solution and where the wage satisfies a no-

shirking condition. In this appendix, we argue that the presence of both bargaining and the need to motivate workers results in a wage which is the maximum of the wages generated under the two hypotheses. In a nonstationary environment, the equilibrium wage would be based on looking ahead, where in each future period the wage would be the maximum of the two approaches. This is similar to the argument that has been made by Kenneth Binmore (1983) and Avner Shaked and John Sutton (1984) when an outside option is available.

This result can be seen from both the axiomatic formulation of the Nash wage and the noncooperative bargaining approach. The equation used above for the Nash wage comes from selecting a wage to maximize the product of the gains from beginning employment for the firm and the worker. Consider what happens if it is known that the worker will shirk (do no work) if the wage is below some critical level. In this case the gain to the firm from hiring the worker is negative. Thus any wage that violates the no-shirking condition does not maximize the product. If the Nash wage is above the critical value, it does maximize the value. If the Nash wage is below the critical value then the efficiency wage maximizes the product since, ignoring shirking, the product is quasiconcave in the wage.

We proceed by describing a bargaining game. Assume that the firm moves first, proposing a wage, w_1. The worker has three options: accept the wage and do not shirk, accept the wage and shirk, reject the wage. If the worker rejects the wage, the worker proposes the next wage, w_2. The firm might accept or reject this wage. If the firm accepts the wage, the worker then gets to choose whether to shirk or not. If the firm rejects the wage, the firm gets to make the next proposal. At any time there is a probability that negotiations will be

exogenously broken off, requiring each party to wait for the next potential partner with whom to take up negotiations. The value of the position in the event of broken negotiations is the status quo point for the Nash solution.

The last move in any sequence which results in a contract is for the worker to decide whether or not to shirk. This decision is made by comparing the wage with the wage coming from the no-shirking condition. This makes it clear that the firm will never offer a wage that is accepted which is below the efficiency wage, and that the value of a contract to the firm and worker is based on production without shirking. We assume that a contract at the no-shirking wage is preferable to the firm to having no contract. The heart of the argument is that if the Nash wage exceeds the efficiency wage, it remains the solution to the bargaining game; if the Nash wage is not above the efficiency wage, there is no internal solution to the bargaining problem, and the efficiency wage is offered and accepted at the first stage.

Since this is a stationary game, we can follow the procedure in Avner Shaked and John Sutton (1984), where the parties understand that the game at step three has the same value as the game at step one. Let K be the value to the worker of being at step 3. At step 3 the value to the firm is W-K, where W is the combined value to the firm and worker from an agreement which results in work without shirking (a value which is independent of the wage). At step 2, waiting until step 3 is worth $\{pW_v+(1-p)(W-K)\}$ to the firm, where p is the probability of an exogenous breakdown in negotiations and W_v the value of being a vacancy. Thus at step 2, the worker's position is worth $W-\{pW_v+(1-p)(W-K)\}$. This is more than the value which ensures no-shirking. Seen from step 1, waiting until step 2 is worth $pW_u+(1-p)\{W-\{pW_v+(1-p)(W-K)\}\}$ to the worker. The firm will make this

offer, unless it is less than the efficiency wage, in which case the firm offers the efficiency wage. Setting the value of the offer to the worker at step 1 equal to K, we have

$$K=W_u+(W-W_u-W_v)(1-p)/(2-p).$$

As p goes to zero, this is the familiar formula for W_e.

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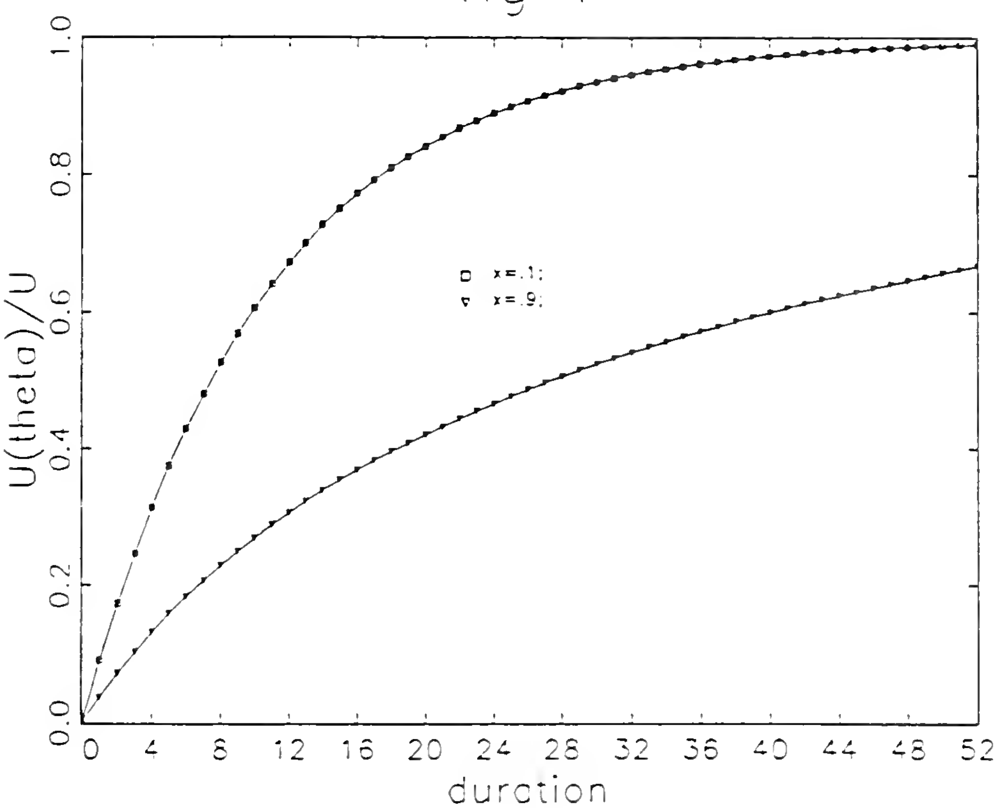
List of Figures

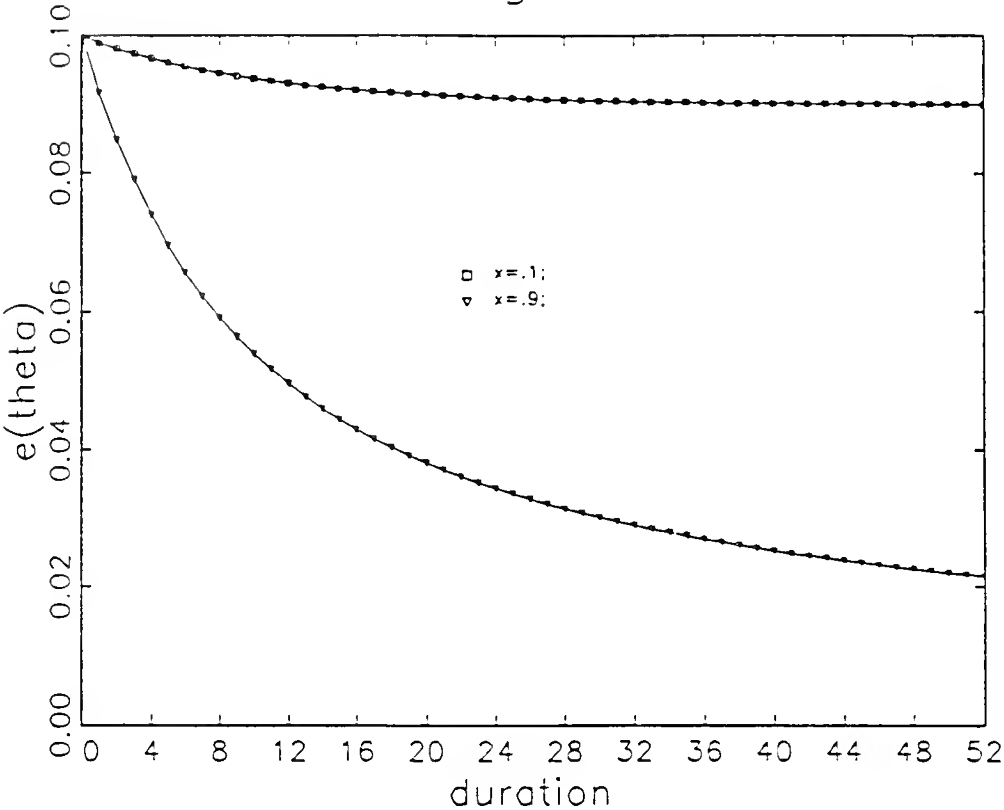
Figure 1. Distribution of unemployment durations.
Figure 2. Exit rates from unemployment as a function of duration.
Figure 3a. Ranking and no-ranking Nash wages as a function of labor market tightness, r = .2%.
Figure 3b. Ranking and no-ranking Nash wages as a function of labor market tightness, r = 1%.
Figure 4a. Unemployment and vacancies in a rapid expansion of the economy.
Figure 4b. Unemployment densities in a rapid expansion of the economy.
Figure 4c. Exit rates in a rapid expansion of the economy.
Figure 4d. Ranking and no-ranking Nash wages in a rapid expansion of the economy.
Figure 5a. Unemployment and vacancies in a slow expansion of the economy.
Figure 5b. Unemployment densities in a slow expansion of the economy.
Figure 5c. Exit rates in a slow expansion of the economy.
Figure 5d. Ranking and no-ranking Nash wages in a slow expansion of the economy.
Figure 6. Ranking and no-ranking efficiency wages in a rapid expansion of the economy.
Figure 7. Ranking and no-ranking efficiency wages in a slow expansion of the economy.

fig 1



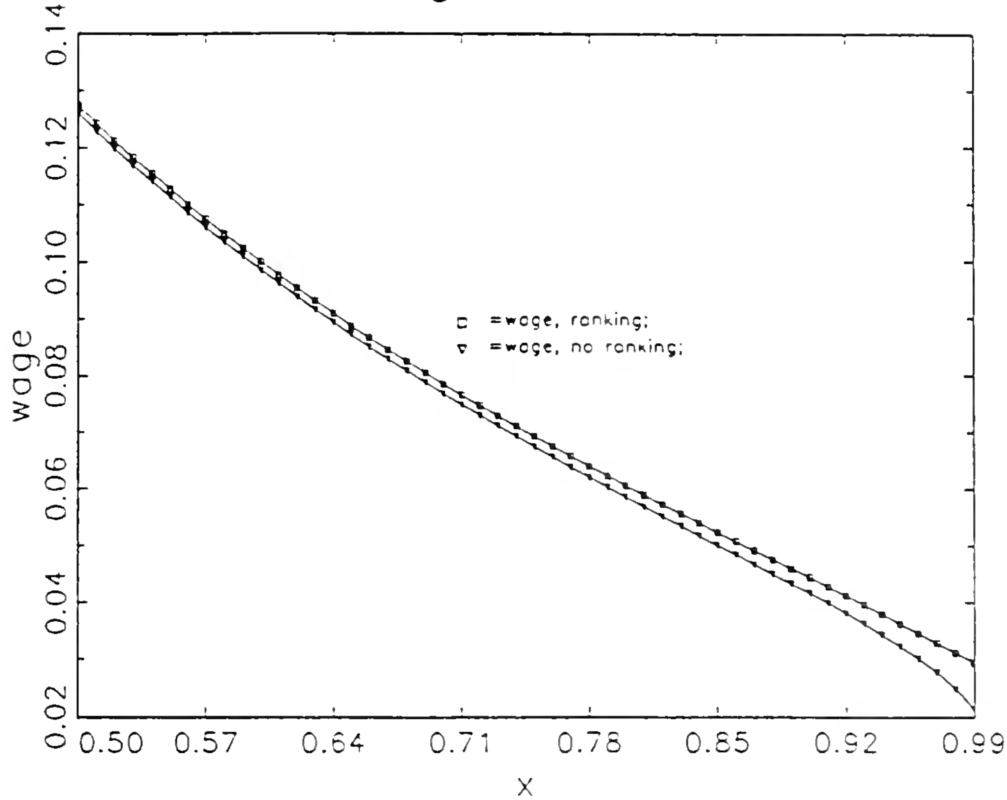
U(theta)/U
duration
x=.1;
x=.9;
0.0
0.2
0.4
0.6
0.8
1.0
0
4
8
12
16
20
24
28
32
36
40
44
48
52

fig 2



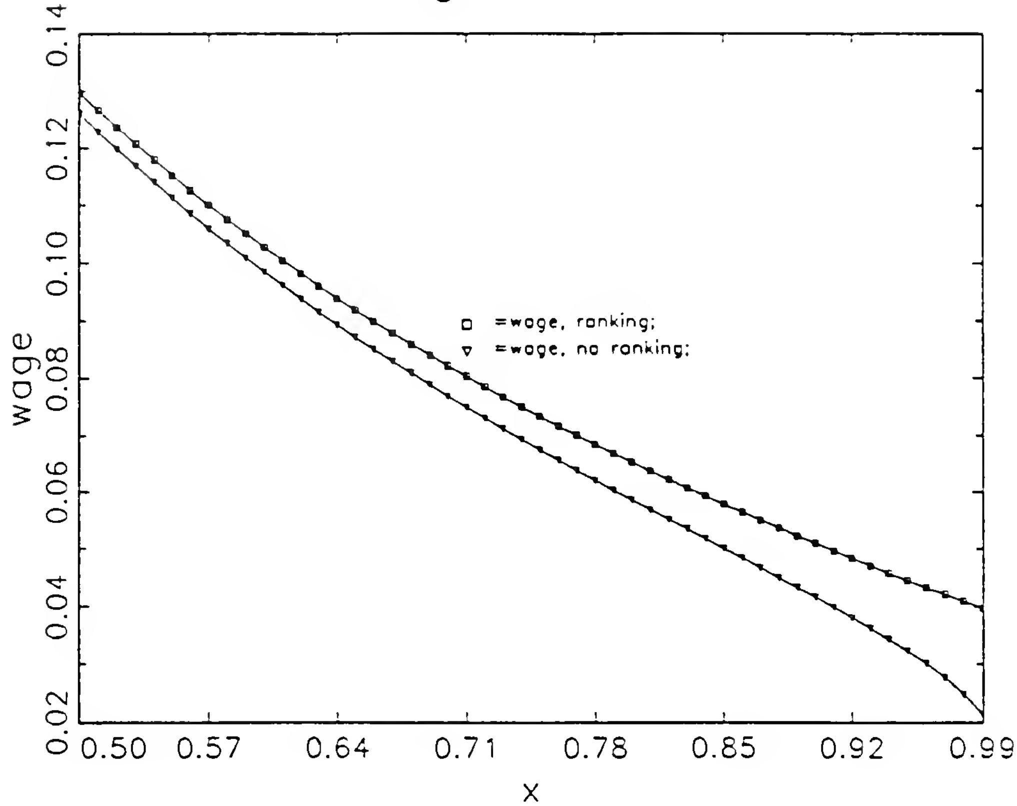
0.10
0.08
0.06
0.04
0.02
0.00
e(theta)
x=.1;
x=.9;
0
4
8
12
16
20
24
28
32
36
40
44
48
52
duration

Figure 3.a



wage
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.50
0.57
0.64
0.71
0.78
0.85
0.92
0.99
X
=wage, ranking;
=wage, no ranking;

Figure 3.b



0.14
0.12
0.10
0.08
0.06
0.04
0.02
wage
=wage, ranking;
=wage, no ranking;
0.50
0.57
0.64
0.71
0.78
0.85
0.92
0.99
x

Figure 4.a

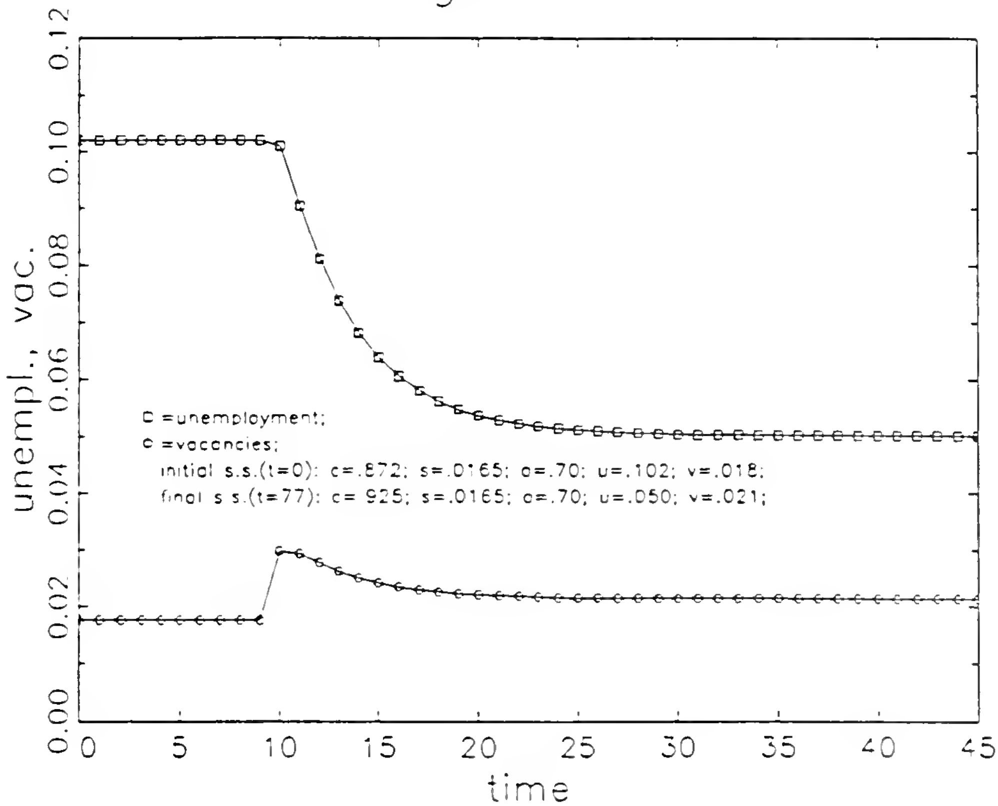
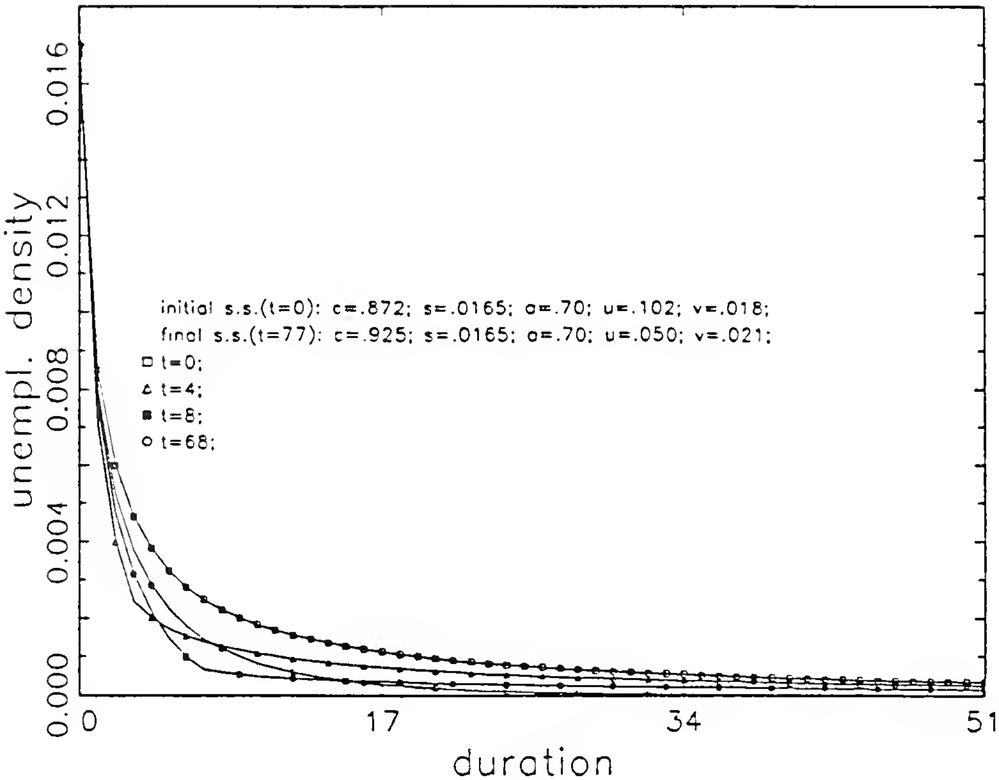
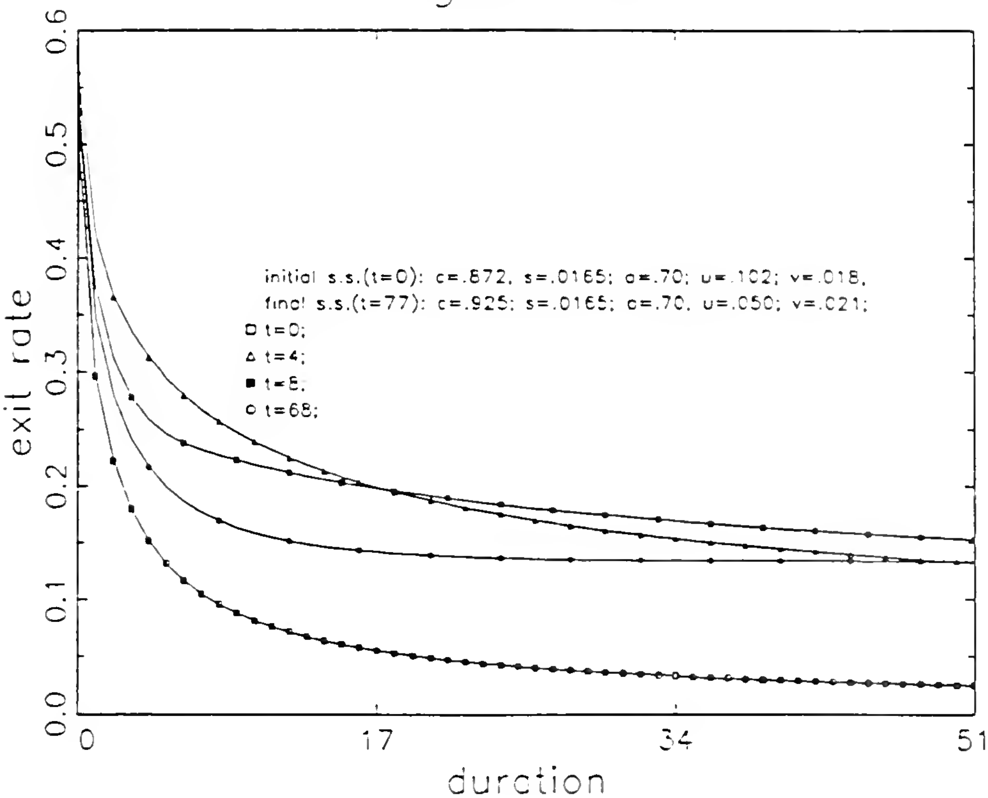

□ =unemployment;
○ =vacancies;
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=77): c=.925; s=.0165; a=.70; u=.050; v=.021;
unempl., vac.
0.00
0.02
0.04
0.06
0.08
0.10
0.12
time
0
5
10
15
20
25
30
35
40
45

Figure 4.b



unempl. density
0.000
0.004
0.008
0.012
0.016
0
17
34
51
duration
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=77): c=.925; s=.0165; a=.70; u=.050; v=.021;
t=0;
t=4;
t=8;
t=68;

Figure 4.c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final s.s.(t=77): c=.925; s=.0165; a=.70, u=.050; v=.021;
t=0;
t=4;
t=8;
t=68;
exit rate
duration
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
17
34
51

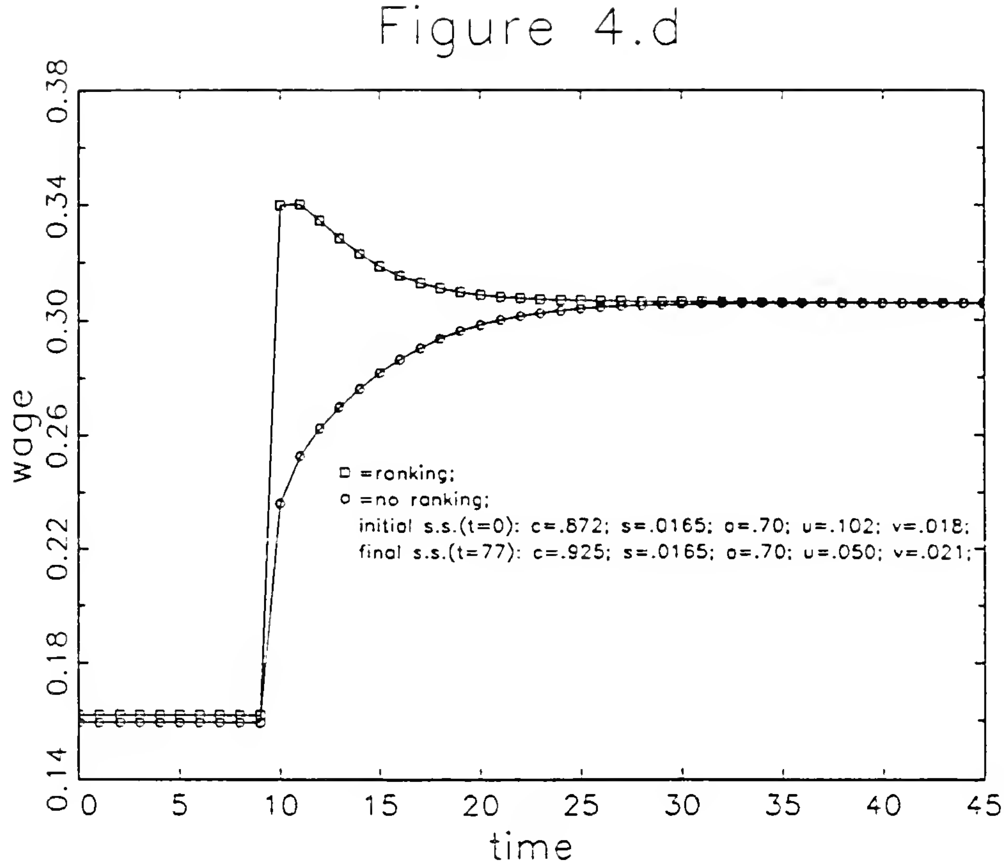
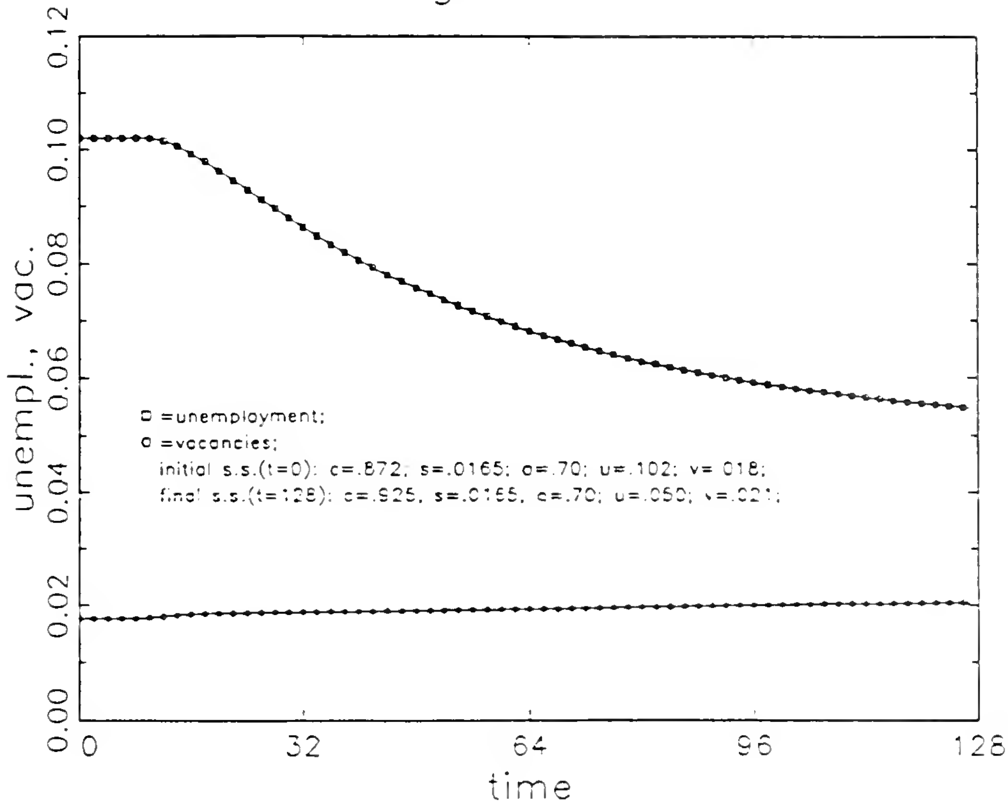


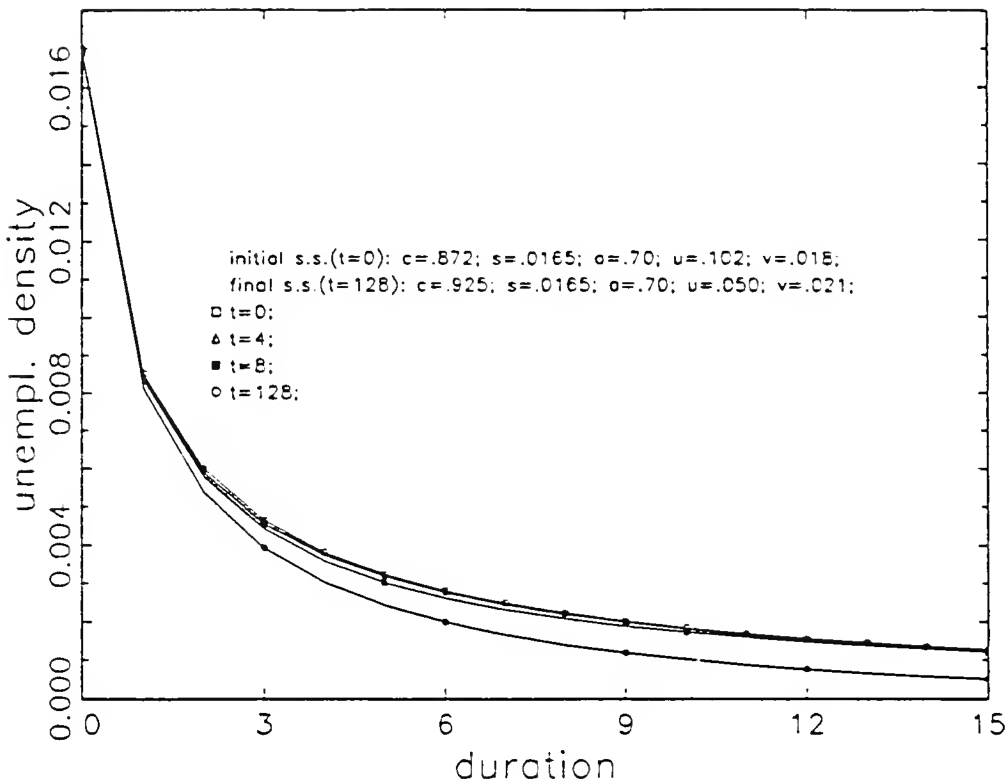
Figure 4.d
wage
time
□ =ranking;
o =no ranking;
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=77): c=.925; s=.0165; a=.70; u=.050; v=.021;

Figure 5.a



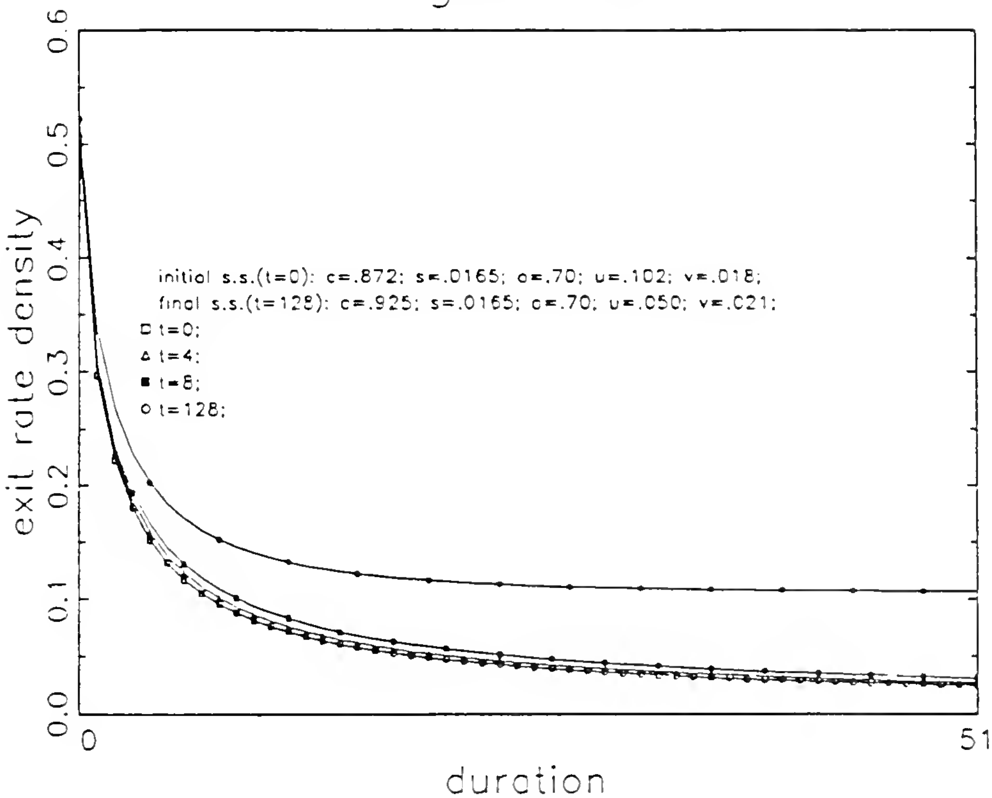
unempl., vac.
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0
32
64
96
128
time
□ =unemployment;
○ =vacancies;
initial s.s.(t=0): c=.872; s=.0165; α=.70; u=.102; v=.018;
final s.s.(t=128): c=.925, s=.0165, α=.70; u=.050; v=.021;

Figure 5.b



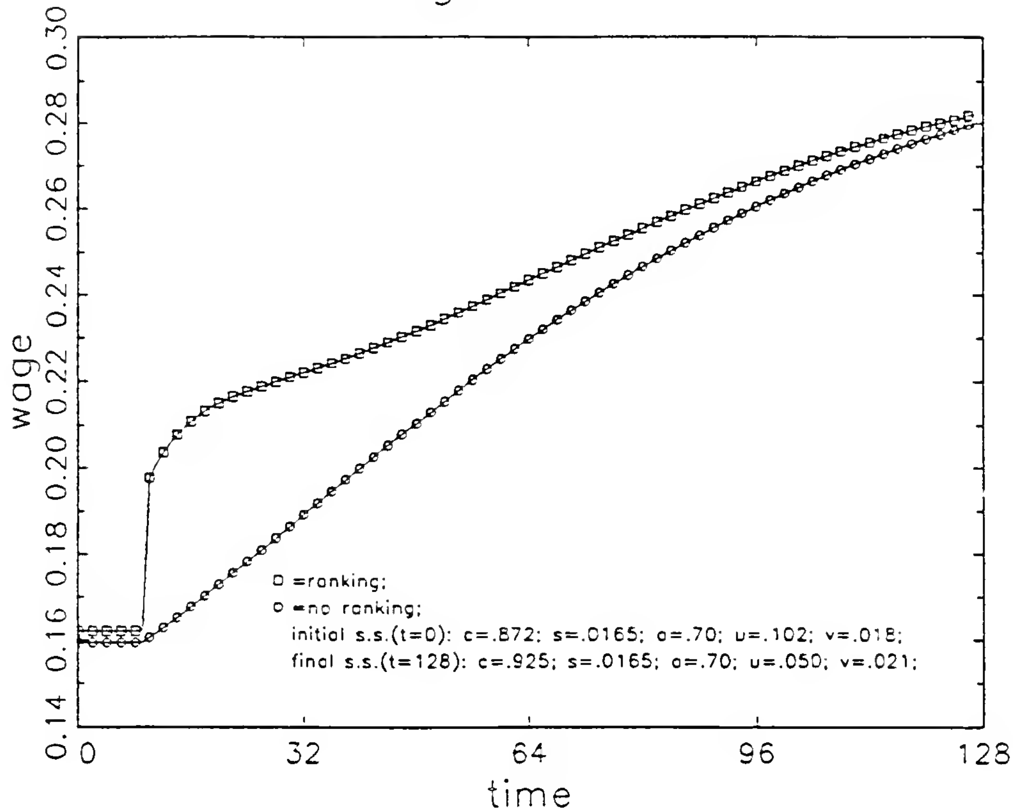
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=128): c=.925; s=.0165; a=.70; u=.050; v=.021;
t=0;
t=4;
t=8;
t=128;
unempl. density
0.000
0.004
0.008
0.012
0.016
duration
0
3
6
9
12
15

Figure 5.c



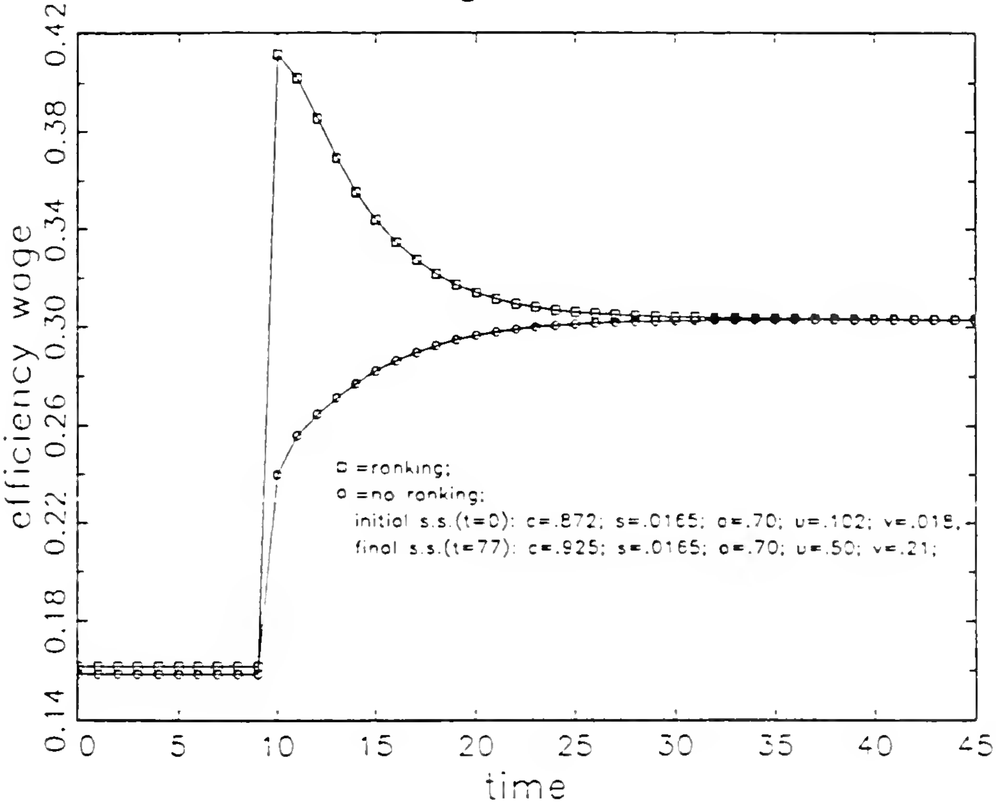
0.6
0.5
0.4
0.3
0.2
0.1
0.0
exit rate density
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=128): c=.925; s=.0165; a=.70; u=.050; v=.021;
t=0;
t=4;
t=8;
t=128;
0
51
duration

Figure 5.d



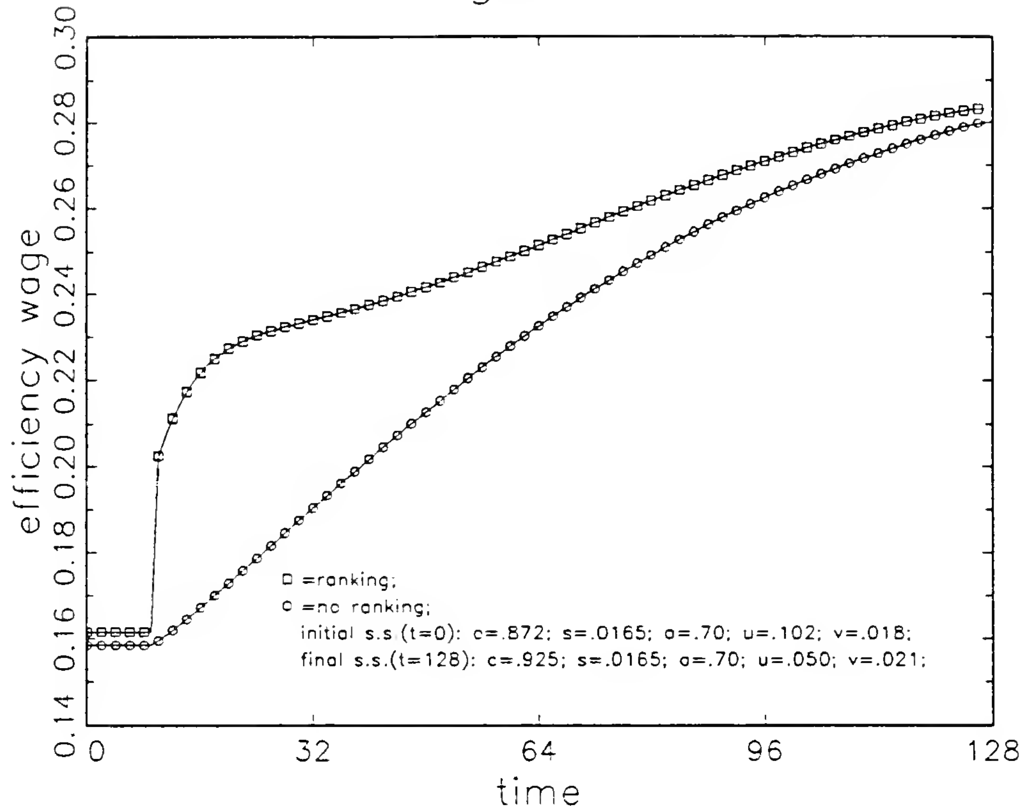
0.14
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
wage
0
32
64
96
128
time
□ =ranking;
○ =no ranking;
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=128): c=.925; s=.0165; a=.70; u=.050; v=.021;

Figure 6



efficiency wage
0.14
0.18
0.22
0.26
0.30
0.34
0.38
0.42
time
0
5
10
15
20
25
30
35
40
45
□ =ronking;
o =no ronking;
initiol s.s.(t=0): c=.872; s=.0165; o=.70; u=.102; v=.018,
finol s.s.(t=77): c=.925; s=.0165; o=.70; u=.50; v=.21;

Figure 7



efficiency wage
0.14
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30
0
32
64
96
128
time
□ =ranking;
○ =no ranking;
initial s.s.(t=0): c=.872; s=.0165; a=.70; u=.102; v=.018;
final s.s.(t=128): c=.925; s=.0165; a=.70; u=.050; v=.021;